Exercises

Portfolio Optimization: Theory and Application
Chapter 9 – High-Order Portfolios

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portfoliooptimizationbook.com

Exercise 9.1: Non-Gaussian return distribution

a. Download market data for one asset.
b. Plot the histograms for different frequencies of returns.
c. Try to fit a Gaussian distribution.
d. Assess the asymmetry as well as the thickness of the tails for these histograms (use Q–Q plots, compute skewness and kurtosis, etc.).

Exercise 9.2: Computation of portfolio sample moments

a. Download market data corresponding to N assets during a period with T observations, $\boldsymbol{r}_1, \ldots, \boldsymbol{r}_T \in \mathbb{R}^N$.
b. Estimate the mean vector, covariance matrix, co-skewness matrix, and co-kurtosis matrix of the data via sample means.
c. Design some portfolio, such as the $1/N$ portfolio, and compute the four moments of the portfolio returns (i.e., mean, variance, skewness, and kurtosis).
d. Additionally, compute the gradient and Hessian of the four portfolio moments.
e. Repeat the whole process for different values of N, while keeping track of the computational cost, and make a final plot of complexity vs. N.

Exercise 9.3: Comparison of nonparametric, structured, and parametric moments

a. Download market data corresponding to N assets during a period with T observations, $\boldsymbol{r}_1, \ldots, \boldsymbol{r}_T \in \mathbb{R}^N$.
b. Design some portfolio, such as the $1/N$ portfolio.
c. Estimate the mean, variance, skewness, and kurtosis of the portfolio returns in the following

ways:

- nonparametric moments: via a direct sample mean estimation of the mean vector, covariance matrix, co-skewness matrix, and co-kurtosis matrix;
- structured moments: via fitting a single market-factor model to the returns;
- parametric moments: via fitting a multivariate skew t distribution to the returns.

d. Repeat the whole process for different values of N, while keeping track of the computational cost, and make a final plot of complexity vs. N.

Exercise 9.4: Sanity check of parametric moment expressions

a. Generate synthetic data according to a multivariate skew t distribution.
b. Design some portfolio, such as the $1/N$ portfolio.
c. Estimate the mean, variance, skewness, and kurtosis of the portfolio returns in the following ways:

   - nonparametric moments: first estimate via sample means the mean vector, covariance matrix, co-skewness matrix, and co-kurtosis matrix of the data, then evaluate the portfolio moments (as well as gradients and Hessians);
   - parametric moments: first fit a multivariate skew t distribution to these synthetic returns, then evaluate the moments with the parametric expressions (as well as gradients and Hessians).

d. Compare the nonparametric and parametric estimations.
e. Repeat the whole process for different numbers of data samples T, and make a final plot of estimators vs. T.

Exercise 9.5: L-moments

a. Download market data for one asset.
b. Compute the first four moments (i.e., mean, variance, skewness, and kurtosis) in a rolling-window fashion and plot them over time.
c. Compute the first four L-moments (i.e., L-location, L-scale, L-skewness, and L-kurtosis) in a rolling-window fashion and plot them over time.
d. Try different values for the lookback window and compare the regular moments with the L-moments.

Exercise 9.6: MVSK portfolios

a. Download market data corresponding to N assets during a period with T observations, $\boldsymbol{r}_1, \dots, \boldsymbol{r}_T \in \mathbb{R}^N$.
b. Fit a multivariate skew t distribution to the data.
c. Design a traditional mean–variance portfolio.
d. Design a high-order MVSK portfolio.
e. Compare their performance. Try to obtain a clear performance improvement via the introduction of higher orders.

Exercise 9.7: Portfolio tilting

a. Download market data corresponding to N assets during a period with T observations, $\boldsymbol{r}_1, \dots, \boldsymbol{r}_T \in \mathbb{R}^N$.
b. Fit a multivariate skew t distribution to the data.
c. Design some portfolio as a reference.
d. Use the portfolio tilting formulation to improve the reference portfolio.
e. Compare their performance. Try to obtain a clear performance improvement via tilting.